Name ________________________ ID3 _ _ _

ECS 315: Probability and Random Processes **2019/1**

HW 11 — Due: November 21, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 2 pages.

(b) (1 pt) Hard-copies are distributed in class. Original pdf file can be downloaded from the course website. Work and write your answers **directly on the provided hardcopy/file** (not on other blank sheet(s) of paper).

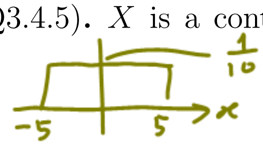
(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page. Furthermore, for online submission, your file name should start with your 10-digit student ID, followed by a space, the course code, a space, and the assignment number: "5565242231 315 HW10.pdf"

(d) (8 pt) It is important that you try to solve all problems.

(e) Late submission will be heavily penalized.

Problem 1 (Yates and Goodman, 2005, Q3.4.5). X is a continuous uniform RV on the interval $(-5, 5)$.

a b



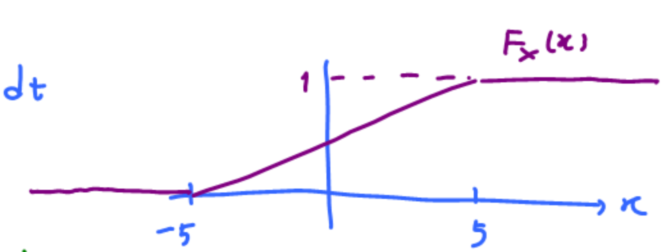
(a) What is its pdf $f_X(x)$?

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a<x<b, \\ 0, & \text{otherwi-} \end{cases} = \begin{cases} \frac{1}{10}, & -5<x<5. \\ 0, & \text{otherwise} \end{cases}$$

(b) What is its cdf $F_X(x)$?

$$P[X \le x] = \int_{-\infty}^{x} f_X(t)\,dt$$

$$F_X(x) = \begin{cases} 0, & x<-5 \\ \frac{1}{10}(x+5), & -5 \le x \le 5 \\ 1, & x>5 \end{cases}$$



(c) What is $\mathbb{E}[X]$?

$= \sum_x x\, p_X(x)$ discrete

$$= \int_{-\infty}^{\infty} x f_X(x)\,dx \quad \text{cont.}$$

$$= \int_{-5}^{5} x \frac{1}{10}\,dx = \frac{1}{10}\frac{x^2}{2}\Big|_{-5}^{5} = 0$$

(d) What is $\mathbb{E}[X^5]$?

$$= \int_{-5}^{5} x^5 \frac{1}{10} dx = \frac{1}{10} \left. \frac{x^6}{6} \right|_{-5}^{5} = 0$$

(e) What is $\mathbb{E}\left[e^X\right]$?

$$\mathbb{E}[e^x] = \int_{-\infty}^{\infty} e^x f_X(x)\, dx = \int_{-5}^{5} e^x \frac{1}{10} dx = \frac{1}{10} \left. e^x \right|_{-5}^{5} = \frac{e^5 - e^{-5}}{10} \approx ?$$

Problem 2 (Randomly Phased Sinusoid). Suppose Θ is a uniform random variable on the interval $(0, 2\pi)$.

$$f_\Theta(\theta) = \begin{cases} 1/2\pi, & 0 < \theta < 2\pi, \\ 0, & \text{otherwise} \end{cases}$$

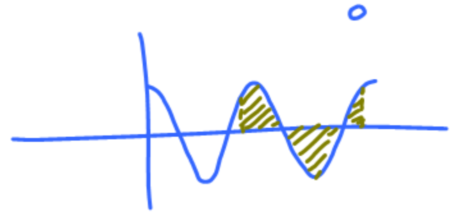
(a) Consider another random variable X defined by

$$X = 5\cos(7t + \Theta) = g(\Theta)$$

where t is some constant. Find $\mathbb{E}[X]$.

$$= \int_{-\infty}^{\infty} g(\theta) f_\Theta(\theta)\, d\theta = \int_0^{2\pi} 5\cos(7t+\theta) \frac{1}{2\pi}\, d\theta = \frac{5}{2\pi} \underbrace{\int_0^{2\pi} \cos(7t+\theta)\, d\theta}_{0}$$

$$= 0$$



(b) Consider another random variable Y defined by

$$Y = 5\cos(7t_1 + \Theta) \times 5\cos(7t_2 + \Theta) = g(\Theta)$$

$$\int_0^{2\pi} c\, d\theta = c(2\pi)$$

where t_1 and t_2 are some constants. Find $\mathbb{E}[Y]$.

$$= \int_{-\infty}^{\infty} g(\theta) f_\Theta(\theta)\, d\theta = \int_0^{2\pi} 5\cos(\overbrace{7t_1+\theta}^{A}) \times 5\cos(\overbrace{7t_2+\theta}^{B}) \frac{1}{2\pi}\, d\theta$$

$$= \frac{25}{2\pi} \times \frac{1}{2} \int_0^{2\pi} \cos(7t_1 + 7t_2 + 2\theta) + \cos(7t_1 - 7t_2)\, d\theta = \frac{25}{4\pi}\left(0 + 2\pi\cos(7t_1 - 7t_2)\right)$$

$$= \frac{25}{2}\cos(7(t_1 - t_2))$$

$$\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$